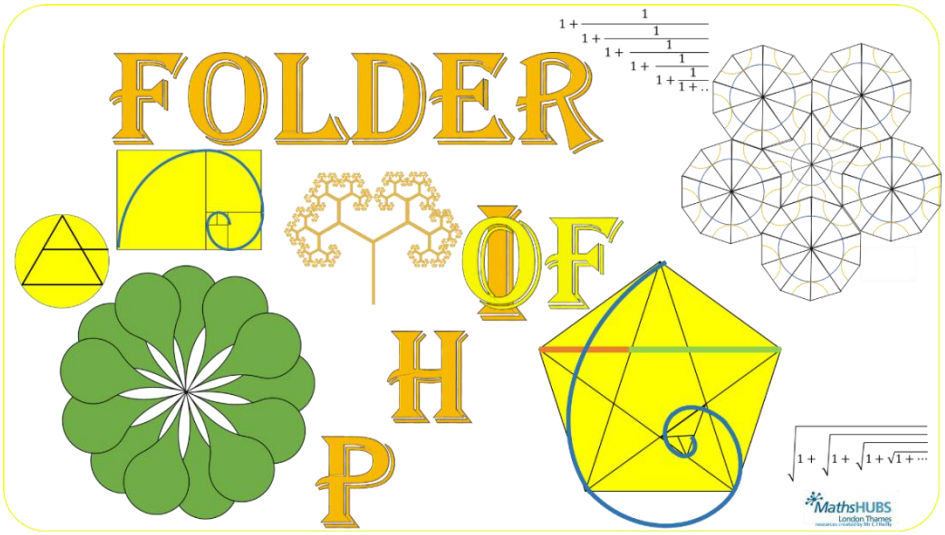

Dear teacher

Why did the 15 lessons on the Golden Ratio begin? As with most adventures my quest began in the classroom. I was teaching surds and a familiar question popped up "when is this ever going to be useful?" My response was to point out that the paper the students were writing on was A4, which has a ratio related to root two, and I then began to talk about the Golden Ratio. I received a second question "the Golden Ratio - what is that?". I made my best attempt to explain the different topics areas linked to the Golden Ratio but found myself constrained to give full justice to the concept of the Golden Ratio. At that point I asked myself "why the student had never encountered the joy of investigating the Golden Ratio before" and "why had they not experienced the appropriate learning environment to develop conceptual understanding between topics".

These are some brief notes on how to make the most of the resources in the "folder of Phi". I have additionally given some commentary regarding the structure of the lessons created, how my own pedagogy has developed. I have also added useful web links and suggested further reading. At the end of the document and I have listed all the lessons created. I hope that this document will be helpful to you.

I began reading as much as I could on the Golden Ratio. I found a variety of topics that could be accessible and challenging to students who are in KS2, KS3 and KS4. Some of the topics I researched on the Golden Ratio had the potential to stretch students all the way up to and beyond the final year of A-level.

My next train of thought was that the vast range of topics related to the Golden Ratio needed to be learnt in the classroom, whereby lesson structures would give students problem solving and investigation tasks that would all lead to the discovery of the Golden Ratio. I wanted to create at least one

lesson on the Golden Ratio for every year group from year 6 to year 13, with a focus on conceptual understanding that could develop students' mathematical abilities through the key stages. This is when the "Golden Ratio for all years" was born.

At this point, I decided to put together a workgroup and recruited a number of teachers in order to plan, deliver and collect feedback from the workgroup members, so that participants could then improve lessons based on the Golden Ratio for students in eight consecutive years of education. At the London Thames Mathsub conference, "Today's teaching for tomorrows learning, successful progression through the key stages 1 - 5 in maths" in February 2017 I delivered a workshop titled "Golden ratio for all years". Participants started the workshop by solving several problems, all giving an answer of Phi. I spoke of how the "Golden Ratio" can be found by solving problems in a wide variety of mathematical topics.

"Each member of the group shared their findings from their independent research, and we were able to discuss and adapt the ideas with the grand scheme of the project in mind. Building the planned lessons into each year group felt very constructive and purposeful."

Dimitri Shaw Mathshub workgroup participant

The benefits of the "Golden Ratio for all years" workgroup was that not only does it support transition and encourages feeder schools and secondary schools to work together but it also allows students to foster an ambition from an early age to reach A level mathematics and to take all they have learnt into further life. Students would receive their "folder of Phi" in year 6 so that they could keep all of their work from the lessons on the Golden Ratio that they would participate in; with the additional benefit of giving them an aspiration of completing the folder when they study mathematics at A level.

Several themes ran through the planned lessons from the "Golden Ratio for all years" workgroup, for example every lesson included a "linking the learning" Fig1 slide at the end to raise the conceptual understanding between the topics and to develop the students' desire to continue learning so that they could complete the next lesson on the "folder of Phi". I really wanted students to be keen, almost to the degree of impatience in wishing to answer the next question that mathematics throws at us.

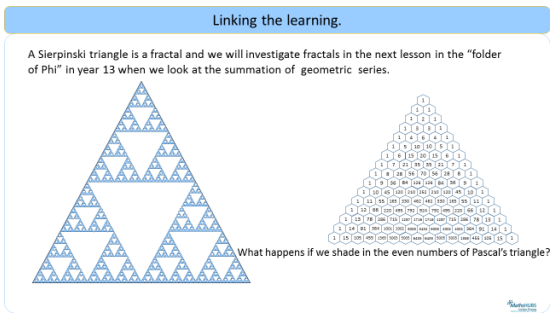

Fig 1

I also developed my own teaching practice through my own participation in the workgroup. I wondered whether I should go back and tweak some of my lessons following developments in my own pedagogy. However, on reflection I thought it is important to relate my own learning journey so that perhaps it can help other teachers improve their pedagogy.

The lessons on surds Fig2 was one of my first- I have differentiated tasks but essentially, it is fluency followed by more stretching fluency.

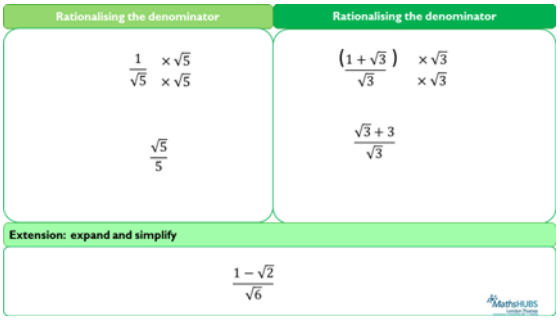

Fig2

We can see that I make the "challenge" style of questions in one of the earlier slides the main task in one of the later ones Fig3. Students were stretched and I directed the rapid graspers to the challenge task.

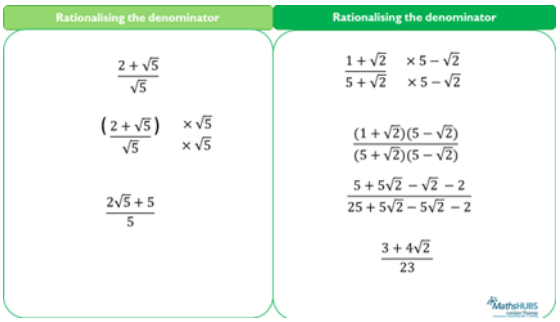

Fig3

However, what about the problem solving and reasoning questions?

To this end, I developed my Pythagoras lessons- one side fluency and one side problem solving Fig4.

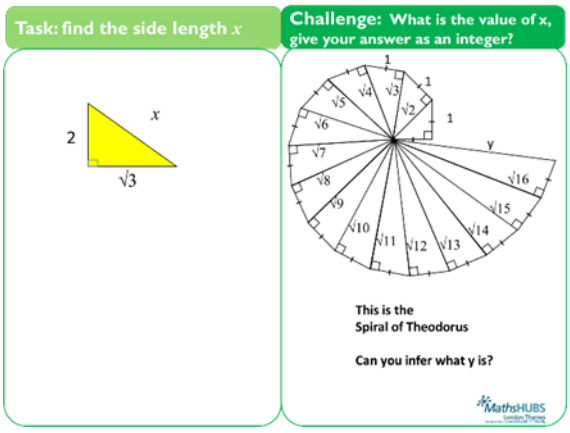

Fig4

There is no link to the questions above other than the fact they are both questions requiring the use of Pythagoras Theorem and this realisation then gave me my next "teaching breakthrough".

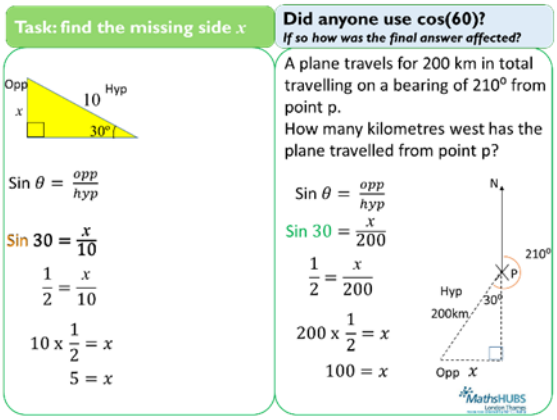

Fig5

We have problem solving and fluency shown above Fig5, appropriate stretch is present and everyone can access the work. As with the Pythagoras questions shown previously, these two slides also ask different questions but still require the same fluency.

Both questions involve sin (30) what is the advantages of this? The real benefits of my "teaching breakthrough" are that you can get everyone to learn first by looking at the fluency and subsequently focus on the challenge task. If a student did the challenge, they still need the understanding from the fluency question to find the exact value of sin (30). Students who did the fluency can now see how they can push onto the challenge questions and to realise that they can use the mathematical skills they have learnt in a problem-solving context. If students can realise "I can do all the mathematics required in this question" you can help support them to do the problem-solving question in the next slide rather than stick with the fluency. My "teaching breakthrough" improved the efficiency of the lesson structure, when I go through the challenge question I can just simply use the exact value of sin (30) that we have just found from the fluency task.

I have also added in a reflection (seen in the top right blue tab at the end of the animation) to allow students to contribute alternative methods to answer the same questions.

There were a number of investigation lessons- the tasks are low threshold and high ceiling. The investigations range from the year 7 "business game", leading to an understanding of tessellation through to the year 11 Tower of Tanoi which can be reviewed again at A level thanks to the "linking the learning" slide. The lessons fully engage the students and this allows them to turn the pictorial understanding into deeper mathematical understanding.

"The lesson on tessellation is going down a treat! Seriously a great session"

Nishma Mehta response after she delivered the year 7 tessellation lesson.
Nishma Mehta has transition responsibilities at her academy between primary and secondary.

Ryan Lee made the year 6 transition lesson where students investigated how to arrange detached and semidetached houses onto various sizes of plots.

"In making my lesson I did not consider the pace at which my lesson would go. Being able to show Chris and get feedback after I told him it went too quickly, I realised that through giving too much assistance I took away the challenge of my lesson. This idea in itself, purposefully restricting any help has made its way into my teaching now as I have begun to value the idea of students needing to independently think and apply what they have been taught."

Ryan Lee Mathshub workgroup participant

Ryan's lesson concept was brilliant. I had not encountered this investigation before but the lesson structure needed some modifications. There was a fine balance between giving the students enough guidance so that no one would become lost, whilst at the same time giving the students enough freedom to explore. My suggestion was to make clear instructions at the start of the lesson. The plots of land are a model and models have limitations. Examples need to demonstrate at the start of the lesson of what will and will not work within a given model. The students need to buy into the benefits of creating models, such as the ability to order information, to make connections between different pieces of information and then to have the ability to make predictions.

As the students begin to explore the model in an investigation lesson I believe the teacher interventions or restraint of them is key. When does the teacher extract from the students how to work systematically? When does the teacher show an example of tabulated results? Often one of the rapid graspers within the lesson will find these revelations independently and the correct concepts can be communicated. Questioning is critical and the ability to let the students see the next paths of enquiry stemming from that point is a magic moment within the classroom.

"Students were very engaged in the house investigation. Students really liked the way that the house project produced the Fibonacci sequence they could articulate how to produce the next term in the sequence."

Lead practitioner Marc Naylor's response to delivering the year 6 transition lesson on the number of combinations of building semidetached and detached houses onto a given number of plots.

The opportunity to talk to students about situations that relate to themselves can really stimulate deep learning. From a simple question on "what are the varieties of accommodation that we can live in?" to "how many different combinations of houses could there be on the street you live on?" and seeing

the reaction on their faces of "that many!" is why I personally got into teaching.

I would advise teachers to reflect on the differences between tasks that develop fluency, problem solving and investigation skills, the benefits of delivering that content and the optimal way of doing this.

I hope that you try out the variety of lessons on the "folder of Phi" and see which lesson structure matches how your students make the greatest progress. Please modify the lessons so that they will be an improvement for the students that you teach.

Christopher Reilly

Mathshub "Golden Ratio for all years" workgroup leader

Web links:

Link to the Golden Ratio "folder of Phi" of the Mathshub lessons
https://www.londonthamesmathshub.com/golden-ratio

Vi Hart has made some great vidoes- play these to your students.
https://www.youtube.com/watch?v=ahXIMUkSXX0&safe=active

Comprehensive website on all you need to know about Fibonacci and the Golden Ratio.
http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibpuzzles.html

Tower of Tanoi interactive game- useful to demonstrate and get the year 11 investigation started.
https://www.mathsisfun.com/games/towerofhanoi.html

In case you want any extra material for the last lesson the "Golden Search"- I would not be surprised if some students starting a degree found the material here challenging. Thank you Dimitri Shaw for this lesson- best title of any lesson that I have some across.
http://mathforcollege.com/nm/mws/gen/09opt/mws_gen_opt_txt_goldensearch.pdf

Looks at art and the Golden Ratio
http://publicism.info/science/golden/7.html

Further reading.

The Golden Ratio: The Story of Phi, the World's Most Astonishing Number
Mario Livio

A fanstastic read- and very comprehensive.

Pascal's triangle 2nd edition
Thomas Green

Another great book with lots of great teaching ideas on Pascal's triangle.

Numercon: A journey through the Hidden lives of numbers
Marianne Freiberger

If Phi has raised your interest in looking at one number in depth then read this book a great starter on looking at all the important numbers in Mathematics.

Acknowledgements

I would like to thank all of the hard work for the members of the work group that created the lessons.

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Lessons and order in which to teach them:

Year 6 House investigation
Year 7 Tessellation
Year 8 Construction
Year 9 Similar shapes
Year 10 Surds part 1
Year 10 Surds part 2
Year 10 Trigonometry and Pythagoras exact values Part 1
Year 10 Trigonometry and Pythagoras exact values Part 2
Year 11 Circle theorems
Year 11 Iterative process part 1 non calculator
Year 11 Iterative process part 2 calculator
Year 11 Quadratic revision
Year 12 Binomial expansion
Year 13 Geometric sum
Year 13+ One dimensional optimisation problem

Investigation lesson skills working systematically, ordering information, analysing findings and applying predictions

Fluency learnt feeds into problem solving questions

Raising standards of Mathematics used to tackle problem stemming from the same task